Chapter 9: solve problems:
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PROBLEM 9-1

Statement: A 22-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 4. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 22$ Diametral pitch $p_d := 4 \cdot in^{-1}$

Solution: See Table 9-1 and Mathcad file P0901.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := \dfrac{N}{p_d}$	$d = 5.5000\,in$
Circular pitch	$p_c := \dfrac{\pi}{p_d}$	$p_c = 0.7854\,in$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := \dfrac{1.0000}{p_d}$	$a = 0.2500\,in$
Dedendum	$b := \dfrac{1.2500}{p_d}$	$b = 0.3125\,in$
Tooth thickness	$t := 0.5 \cdot p_c$	$t = 0.3927\,in$
Clearance	$c := \dfrac{0.2500}{p_d}$	$c = 0.0625\,in$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-5

Statement: A spur gearset must have pitch diameters of 4.5 and 12 in. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference or undercutting and what are the number of teeth on each gear that result from using this diametral pitch? Assume that both gears are cut with a hob.

a. For a 20-deg pressure angle
b. For a 25-deg pressure angle

Given: Pitch diameters: $d_1 := 4.5 \cdot in$ $\qquad d_2 := 12 \cdot in$

Solution: See Table 9-4 and Mathcad file P0905.

1. To avoid undercutting, use the minimum tooth numbers given in Table 9-4b.

a. Pressure angle of 20 deg.

$$N_{min} := 21 \qquad p_{dmin} := \frac{N_{min}}{d_1} \qquad p_{dmin} = 4.667\ in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 5. But, since the pinion pitch diameter is not an integer, using 5 would result in a noninteger number of teeth. Therefore, we must go to the next larger (even) pitch (smaller tooth size) of $p_d := 6 \cdot in^{-1}$. The resulting tooth numbers are:

$$N_1 := p_d \cdot d_1 \qquad N_1 = 27 \qquad N_2 := p_d \cdot d_2 \qquad N_2 = 72$$

b. Pressure angle of 25 deg

$$N_{min} := 14 \qquad p_{dmin} := \frac{N_{min}}{d_1} \qquad p_{dmin} = 3.111\ in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 4. $p_d := 4 \cdot in^{-1}$. The resulting tooth numbers are:

$$N_1 := p_d \cdot d_1 \qquad N_1 = 18 \qquad N_2 := p_d \cdot d_2 \qquad N_2 = 48$$

PROBLEM 9-6

Statement: Design a simple, spur gear train for a ratio of -9:1 and a diametral pitch of 8. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 9$ Diametral pitch $p_d := 8 \cdot in^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth (see Table 9-4b) for a 20-deg pressure angle.

Design Choice: Pressure angle $\phi := 20 \cdot deg$

Solution: See Mathcad file P0906.

1. From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 9. therefore, let the number of teeth on the pinion be

$$N_p := 17 \qquad \text{and} \qquad N_g := m_G \cdot N_p \qquad N_g = 153$$

2. Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \qquad d_p = 2.1250\,in \qquad d_g := \frac{N_g}{p_d} \qquad d_g = 19.1250\,in$$

3. Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \qquad r_p = 1.0625\,in \qquad r_g := 0.5 \cdot d_g \qquad r_g = 9.5625\,in$$

$$a_p := \frac{1}{p_d} \qquad a_p = 0.1250\,in \qquad a_g := \frac{1}{p_d} \qquad a_g = 0.1250\,in$$

Center distance $C := r_p + r_g$ $C = 10.6250\,in$

$$Z := \sqrt{\left(r_p + a_p\right)^2 - \left(r_p \cdot cos(\phi)\right)^2} + \sqrt{\left(r_g + a_g\right)^2 - \left(r_g \cdot cos(\phi)\right)^2} - C \cdot sin(\phi) \qquad Z = 0.6287\,in$$

Contact ratio $m_p := \dfrac{p_d \cdot Z}{\pi \cdot cos(\phi)}$ $m_p = 1.704$

PROBLEM 9-10

Statement: Design a compound, spur gear train for a ratio of -70:1 and diametral pitch of 10. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio: $m_G := 70$ Diametral pitch: $p_d := 10 \cdot in^{-1}$

Solution: See Mathcad file P0910.

1. Since the ratio is negative, we want to have an odd number of stages or an even number with an idler. Let the number of stages be

   Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

   then

$j =$	$r(j) =$
2	8.367
3	4.121
4	2.893

2. Two stages would result in a stage ratio less than 10 but will require an idler, so we will use three stages. The average ratio for three stages is about 21:5. Using a pressure angle of 20 deg, let the stage ratios be

   Stage 1 ratio $r_1 := \dfrac{21}{5}$ Stage 2 ratio $r_2 := \dfrac{20}{5}$ Stage 3 ratio $r_3 := \dfrac{25}{6}$

   and let the driver gears have tooth numbers of

   Tooth number index $i := 2, 3 .. 7$

   $N_2 := 20$ $N_4 := 20$ $N_6 := 18$

   then the driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$ $N_7 := r_3 \cdot N_6$

   $N_3 = 84$ $N_5 = 80$ $N_7 = 75$

   The pitch diameters are: $d_i := \dfrac{N_i}{p_d}$

   Tooth numbers:

$i =$	$\frac{d_i}{in} =$	$i =$	$N_i =$
2	2.0000	2	20
3	8.4000	3	84
4	2.0000	4	20
5	8.0000	5	80
6	1.8000	6	18
7	7.5000	7	75

   Checking the overall gear ratio: $\dfrac{N_3 \cdot N_5 \cdot N_7}{N_2 \cdot N_4 \cdot N_6} = 70.000$

PROBLEM 9-19

Statement: Design a compound, reverted, spur gear transmission that will give two shiftable ratios of +3:1 forward and -4.5:1 reverse with diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 3$ Diametral pitch $p_d := 6 \cdot in^{-1}$

Solution: See Mathcad file P0919.

1. Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 2$ Stage 2 ratio $r_2 := 1.5$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \dfrac{N_3}{N_2}$ $r_2 := \dfrac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 7.500$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 6 \cdot K_{min}$ $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   $K = 45.000$ $N_2 = 15$ $N_4 = 18$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears for the forward train will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 30$ $N_5 := r_2 \cdot N_4$ $N_5 = 27$

   The pitch diameters are: $d_i := \dfrac{N_i}{p_d}$

$i =$	$\dfrac{d_i}{in} =$	$N_i =$
2	2.5000	15
3	5.0000	30
4	3.0000	18
5	4.5000	27

   Checking the overall gear ratio:

   $$\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 3.000$$

5. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

   Stage 1 ratio $r_1 := 2$ Stage 2 ratio $r_2 := 2.25$

6. Let the number of teeth on the reverse stage driver gear be $N_6 := 12$ then the number of teeth on the driven gear will be

   Driven reverse gear $N_7 := r_2 \cdot N_6$ $N_7 = 27$

PROBLEM 9-26a

Statement: Figure P9-2 shows a compound planetary gear train (not to scale). Table P9-2 gives data for gear numbers of teeth and input velocities. For the data in row *a*, find ω_2.

Given: Tooth numbers:

$$N_2 := 50 \qquad N_3 := 25 \qquad N_4 := 45 \qquad N_5 := 30 \qquad N_6 := 40$$

Input velocities: $\omega_6 := 20$ $\qquad \omega_{arm} := -50$

Solution: See Figure P9-2 and Mathcad file P0926a.

1. The formula method will be used in this solution. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be 3 and last be 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_3 - \omega_{arm}} = R$$

2. Calculate R using equation 9.14 and inspection of Figure P9-2.

$$R := \frac{N_3 \cdot N_5}{N_4 \cdot N_6} \qquad\qquad R = 0.4167$$

R is positive in this case because gears 3 and 6 rotate in the same direction.

3. Solve the right-hand equation in step 1 for ω_3.

$$\omega_3 := \frac{\omega_6 - \omega_{arm}}{R} + \omega_{arm} \qquad\qquad \omega_3 = 118.000$$

4. Solve for ω_2 using equation 9.7.

$$\omega_2 := -\frac{N_3}{N_2} \cdot \omega_3 \qquad\qquad \omega_2 = -59.000$$

ω_2 will be in the opposite direction as ω_6.

PROBLEM 9-38

Statement: Figure P9-6*b* shows a differential. Gear A is driven at 10 rpm CCW and gear B is driven CW at 24 rpm. The tooth numbers are indicated in the figure. Find the speed of gear D.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$$N_{A'} := 18 \qquad N_{B'} := 18 \qquad N_C := 18 \qquad N_D := 30$$

Input speeds: $\omega_A := 10 \cdot rpm$ $\qquad \omega_B := -24 \cdot rpm$

Solution: See Figure P9-6*b* and Mathcad file P0938.

1. Determine the speed of the sun gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A' and last be B'. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_{B'} - \omega_{arm}}{\omega_{A'} - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-6*b*.

$$R := \left(-\frac{N_{A'}}{N_{B'}} \right) \qquad R = -1.00000$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_{A'} := \omega_A \qquad \omega_{B'} := \omega_B$$

$$\omega_{arm} := \frac{R \cdot \omega_{A'} - \omega_{B'}}{R - 1} \qquad \omega_{arm} = -7.000\, rpm$$

Gear D is attached to the arm shaft so,

$$\omega_D := \omega_{arm} \qquad \omega_D = -7.00\, rpm$$

PROBLEM 9-57

Statement: A pinion with a 3.000-in pitch diameter is to mesh with a rack. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference?

a. For a 20-deg pressure angle
b. For a 25-deg pressure angle

Given: Pitch diameter: $d := 3.000 \cdot in$

Solution: See Table 9-4 and Mathcad file P0957.

1. Assume thatthe pinion is generated by means other than being cut by a hob.

a. Pressure angle of 20 deg.

$$N_{min} := 18 \qquad p_{dmin} := \frac{N_{min}}{d} \qquad p_{dmin} = 6.000\, in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 6. $p_d := 6 \cdot in^{-1}$.
The resulting number of teeth on the pinion is:

$$N := p_d \cdot d \qquad N = 18$$

b. Pressure angle of 25 deg

$$N_{min} := 12 \qquad p_{dmin} := \frac{N_{min}}{d} \qquad p_{dmin} = 4.000\, in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 4. $p_d := 4 \cdot in^{-1}$.
The resulting number of teeth on the pinion is:

$$N := p_d \cdot d \qquad N = 12$$

PROBLEM 9-64

Statement: Figure P9-35h shows (schematically) a compound epicyclic train. The tooth numbers are 80, 20, 25, and 85 for gears 2, 3, 4, and 5, respectively. Gear 2 is driven at 200 rpm CCW and gear 5 is fixed to ground. Determine the speed and direction of the arm. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_2 := 80 \qquad N_3 := 20 \qquad N_4 := 25 \qquad N_5 := 85$

Input speeds: $\omega_2 := 200\cdot rpm \qquad \omega_5 := 0\cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-35h and Mathcad file P0964.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7*b*.

$$R := \left(\frac{N_2}{N_3}\right)\cdot\left(\frac{N_4}{N_5}\right) \qquad R = 1.17647$$

Solve the right-hand equation above for ω_{arm} with $\omega_5 = 0$.

$$\omega_{arm} := \frac{R}{R-1}\cdot\omega_2 \qquad \omega_{arm} = 1333\,rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \qquad \rho = 1.176$$

3. The combination of $\rho > 1$, shaft 1 fixed, and input to gear 2 corresponds to Case 3 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho\cdot E_0 - 1}{E_0\cdot(\rho - 1)} \qquad \eta = 0.884$$